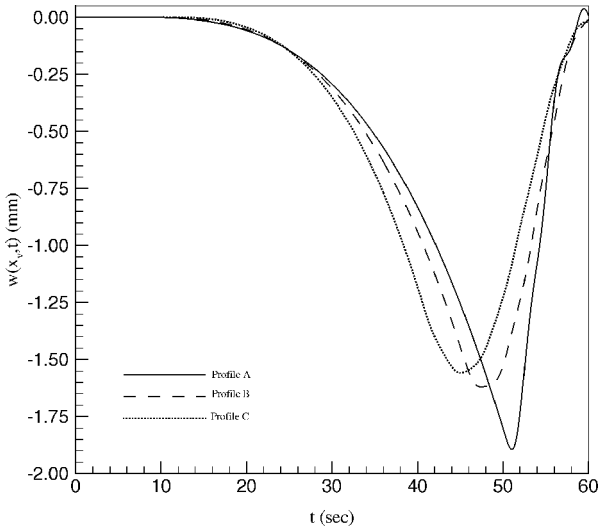

Fig. 2 Deflection under moving mass for various travel profiles.

acceleration contributes to a larger stiffness modulation. Given that profile A displayed the maximum deformation but the least velocity and acceleration, it is probable that, in this example, the role played by velocity in the forcing term dominates the combined stiffening effects of velocity and acceleration.

At a point that is within the constant velocity phase, these three curves cross one another so that now C exhibits the greatest deflection, followed by B, and then A. The deformations in the constant velocity phase indicate that the forcing terms dominate over the stiffening components. The magnitude of the deformations across the profiles is proportional to the magnitude of the constant velocity attained during this phase. The continued increasing deformation exhibited by profiles B and C is a consequence of their longer constant velocity phase during which energy is further added to the system.

The magnitude of the velocity forcing term decreases over the third phase, and the deceleration softens the stiffness modulation. This explains the decreasing deformation experienced with each traversal profile. Hence the reversal in the magnitudes of the deformation such that, during approximately the last 10 s, profile A deforms the most, followed by B, and then C. The deformation of profile B is bounded by the profiles A and C because the velocities and accelerations of the traversing mass are similarly bounded.

Based on the examined profiles, it can be inferred that, using the relative magnitude of the deformation under the traversing mass as a basis for comparison, the travel profile with the minimum constant velocity phase duration is the best choice. Although the profile with the minimum constant velocity phase duration has the maximum constant velocity and hence experiences the greatest forcing, it receives the smallest impulse. The traversing mass acceleration is not modulating the stiffness of the system during this phase because it is zero. The same profile has the least deceleration, which corresponds to the least softening effect, which in turn implies the least deformation.

Conclusions

The system of governing equations for the planar orbiting dynamics of a flexible beam attached to a satellite and traversed by a moving mass show that the velocity and acceleration of the traversing mass act as stiffness modulators. The former appears in a quadratic form and was retained during the simulations because the velocity at which its contribution is negligible is not intuitively evident. The velocity of the traversing mass also contributes to the system forcing term. It is inferred from the simulated travel profiles that it is best, based on the relative magnitudes of the system deformations, to use a travel profile with the shortest constant velocity phase, or equivalently the smallest possible accelerations and decelerations.

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Cyclic Creep of Piezoelectric Polymer Polyvinylidene Fluoride

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Nomenclature

t = time
ε_m = creep strain
ε_v = vibrocreep strain
σ_a = stress amplitude
σ_m = mean stress
ω = frequency

Introduction

In the past few decades a new generation of synthetic piezoelectric polymers has emerged that possess the ability to actively react to changing stimuli as a result of energy conversion from mechanical to electrical and vice versa. Piezoelectric polymer systems have been increasingly integrated in structural design as active elements capable of sensing and responding intelligently to external stimuli. A broad range of applications utilizing such functions include active vibration damping, acoustic suppression, damage detection, shape and position control of compliant structures, and self-inspection of structural integrity.[1,2]

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Currently, polyvinylidene fluoride (PVDF) is the principal commercially available polymer that exhibits strong piezoelectric properties. Typically, PVDF is produced in the form of thin films of thickness ranging from 102×10^{-4} to 762×10^{-3} mm. A thin layer of nickel or aluminum is deposited on both surfaces of the material in order to provide electrical conductivity when an electrical field is applied or to allow measurements of the charge induced by mechanical deformations.

Since the discovery of piezoelectric effects in PVDF by Kawai,[3] the properties of this material have been studied by many investigators. However, research efforts in this regard have been focused primarily on the piezoelectric response of the polymer, whereas the mechanical properties of PVDF have attracted limited attention.[4] The objective of this Note is to provide a summary of research findings regarding the time-dependent response of PVDF thin films under combined static and cyclic loading conditions.

General Properties of PVDF

Polyvinylidene fluoride is a semicrystalline polymer with typical crystallinity of, approximately, 50%. The amorphous phase of the polymer has the properties of a supercooled liquid with the glass transition temperature of about -50°C. The molecular structure of polyvinylidene fluoride consists of a repeated monomer unit $-CF_2-CH_2-$. The atoms are covalently bonded together forming long molecular chains. Piezoelectric properties of PVDF are obtained by subjecting the polymer to an elevated, typically, 130°C temperature and an electric field of up to 100 kV/mm. Stretching and polling of extruded thin sheets of the polymer causes an alignment of its molecular chains in the stretch direction. The material is then cooled down while the poling field is maintained. As a result, the alignment of dipoles is permanently fixed.

The electromechanical properties of PVDF thin films are commonly defined by the constitutive equations of linear piezoelectricity.[5] The theory is based on the assumption that the relations between stress and strain tensors, electric flux density, electric field, and dielectric tensors are linear. Energy dissipation in the material under cyclic loading conditions is neglected. However, experiments indicate that these assumptions have certain limitations. In particular, it is shown that PVDF is a polymer characterized by a nonlinear stress-dependent piezoelectric response.[6] The polymer has also demonstrated creep and relaxation behavior, as well as measurable energy losses when subjected to cyclic loads.[7,8] Within certain limits under static loading conditions, the creep properties of PVDF can be characterized as linearly viscoelastic.[9,10]

Vibrocreep of PVDF

In this study the time-dependent behavior of PVDF has been investigated under the conditions of combined static and cyclic loads. The investigation has been motivated by the empirical evidence that, in general, polymers tend to exhibit measurable changes in their creep response as a result of cyclic loading effects. This phenomenon, known as *vibrocreep*, is usually observed in terms of accelerated creep rates as compared with the creep rates at the respective static loads.[11–14]

The understanding of vibrocreep effects in the behavior of PVDF is of particular practical importance, first, because the polymer is typically employed as an active element in sensors and actuators designed to functions in vibratory environments, and, second, because PVDF thin films can perform only in the tension-tension mode, always requiring static pretension.

In this study thin film 7.62×76.2 mm PVDF samples of 28-μm thickness with two 10-μm silver layers deposited on both surfaces of the film have been tested under superimposed static and cyclic stress conditions defined as $\sigma(t) = \sigma_m + \sigma_a \sin \omega t$. Experiments have been performed in the direction of the aligned molecular chains of the polymer because PVDF thin films have demonstrated greater creep rates in that direction.[9,10] Tests have been conducted at 23°C at three different frequencies, $\omega = (5, 10, \text{and } 20)$ Hz, over a range of mean stresses σ_m and stress amplitudes σ_a, defined in relation to the respective yield stress $\sigma_Y = 45$ MPa, as shown in Table 1.

Table 1 Cyclic test conditions for PVDF thin films

Mean stress, $\sigma_m = n\sigma_Y$	Amplitude, $\sigma_a = m\sigma_Y$			
n	m			
0.30	0.1	0.2	——	——
0.45	0.1	0.2	0.35	0.4
0.60	0.1	0.2	0.40	0.5

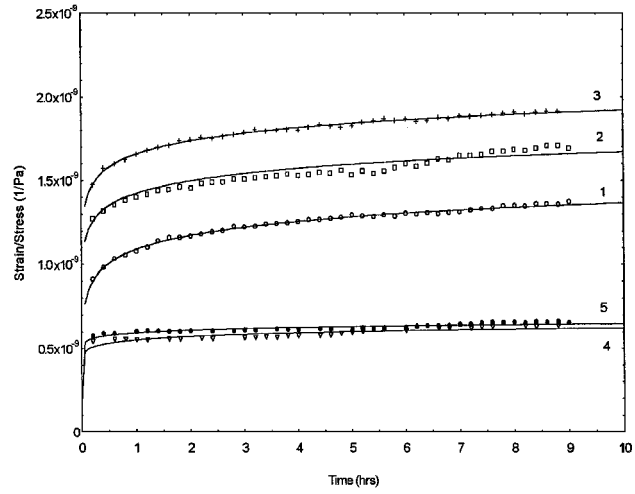

Fig. 1 Vibrocreep of PVDF: frequency effect. Cyclic ($\sigma_m = 0.45\sigma_{Y1}$, $\sigma_a = 0.1\sigma_{Y1}$) 1, $\omega = 5$ Hz; 2, $\omega = 10$ Hz; and 3, $\omega = 20$ Hz. Static 4, $0.45\sigma_{Y1}$; and 5, $0.6\sigma_{Y1}$.

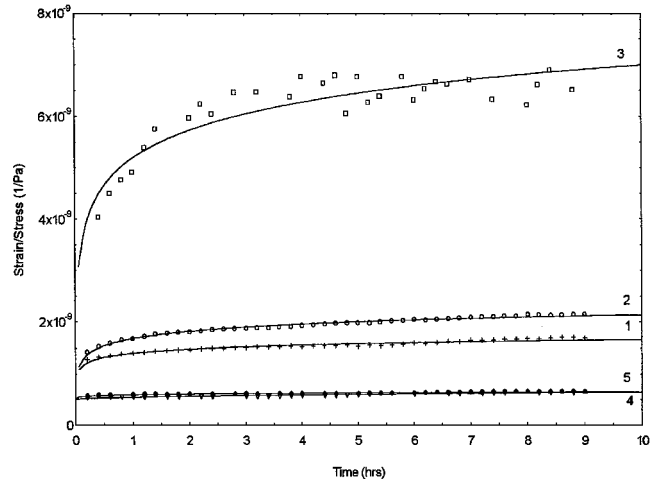

Fig. 2 Vibrocreep of PVDF: amplitude effect. Cyclic ($\sigma_m = 0.45\sigma_{Y1}$, $\omega = 10$ Hz) 1, $\sigma_a = 0.1\sigma_{Y1}$; 2, $\sigma_a = 0.2\sigma_{Y1}$; and 3, $\sigma_a = 0.4\sigma_{Y1}$. Static 4, $0.45\sigma_{Y1}$; and 5, $0.6\sigma_{Y1}$.

Throughout the entire program PVDF samples have been prepared in compliance with ASTM D882-95a standards. At least three to five identical experiments have been performed to ensure reproducible results.

In the study vibrocreep effects have been characterized in terms of the measured vibrocreep strain $\varepsilon_v(t)$ and normalized vibrocreep creep strain $\varepsilon_v(t)/\sigma_m$. The obtained experimental results have been assessed in relation to static creep defined, respectively, in terms of $\varepsilon_m(t)$ or $\varepsilon_m(t)/\sigma_m$, where $\epsilon_m(t)$ denotes time-dependent strain under constant stress σ_m.

Some representative results demonstrating vibrocreep effects depending on three parameters of frequency, mean stress, and stress amplitude are shown in Figs. 1–3. A more complete presentation of the results obtained in the course of this investigation is given in Ref. 15.

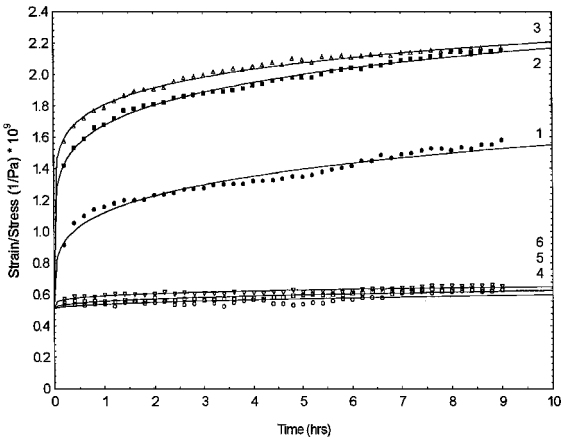

Fig. 3 Vibrocreep of PVDF: mean stress effect. Cyclic ($\sigma_a = 0.2\sigma_{Y1}$, $\omega = 10$ Hz) 1, $\sigma_m = 0.3\sigma_{Y1}$; 2, $\sigma_m = 0.45\sigma_{Y1}$; and 3, $\sigma_m = 0.6\sigma_{Y1}$. Static 4, $0.3\sigma_{Y1}$; 5, $0.45\sigma_{Y1}$; and 6, $0.6\sigma_{Y1}$.

Conclusions

The study of the cyclic creep response of PVDF thin films demonstrates that the polymer tends to exhibit accelerated creep rates under the conditions of superimposed static and cyclic loads. Creep acceleration caused by cyclic loading effects has been observed even in the range of stresses well below the viscoelastic linearity limit. It is clear that the long-term cyclic response of PVDF is essentially nonlinear because it does not represent a simple superposition of the responses to static and fully reversed cyclic loads applied separately.

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